Modifying Children's Tasks into Cognitively Demanding Tasks for Preservice Elementary Teachers

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Tasks & task design

> *"What students learn is largely defined by the tasks they are given."*
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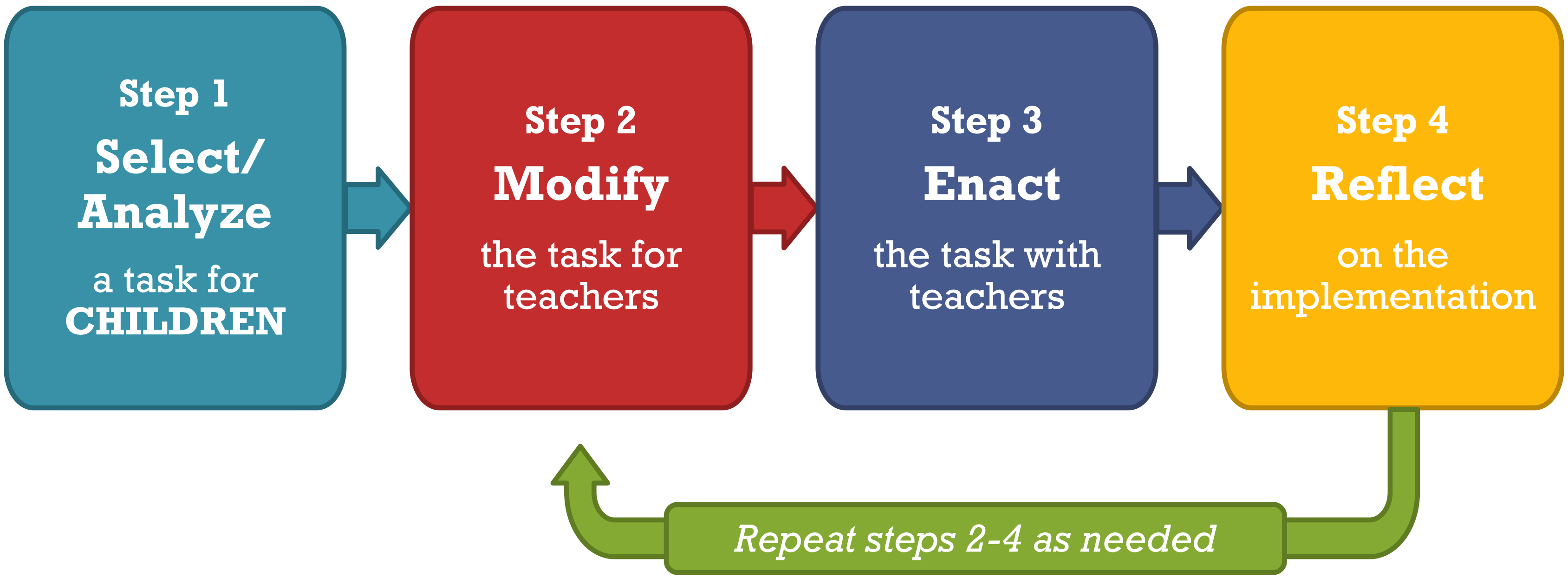
> (Hiebert & Wearne, 1993, p. 395)

- Tasks play a key role in teacher education by providing meaningful interventions to enhance knowledge and practices of teachers (Chapman, 2013)
- Improving the quality of mathematical tasks used with PSTs will improve the overall instruction in K–12 classrooms (Watson & Mason, 2007)

Framework for task development

Tobias, Olanoff, Hillen, Welder, Feldman, & Thanheiser (2014)

Iterative cycle of task design to guide MTEs in utilizing children's tasks in content courses with PSTs



Tobias, J. M., Olanoff, D., Hillen, A., Welder, R. M., Feldman, Z., & Thanheiser, E. (2014). Research-based modifications of elementary school tasks for use in teacher preparation. In K. King (Ed.), *Annual Perspectives in Mathematics Education: Using Research to Improve Instruction* (pp. 181-192). Reston, VA: NCTM.

Task modifications

Cognitive demand refers to the kind of thinking processes involved in solving a task.

(Stein, Smith, Henningsen, & Silver (2009)

- Tasks that are high-level for children are not necessarily high-level for PSTs
- Children's tasks may need to be modified to provide PSTs an appropriate level of challenge

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Children's task: Comparing fractions

Russell, S. J., et al. (2008). *Investigations in number, data, and space student activity book, Grade 5* (2nd ed.), p. 21. Glenview, IL: Scott Foresman.

Which Is Greater? (page 1 of 2)

Solve the problems below and explain or show how you determined the answer.

1. Which is greater? $\frac{7}{10}$ or $\frac{3}{5}$

2. Which is greater? $\frac{7}{8}$ or $\frac{9}{10}$

3. Which is greater? $\frac{4}{3}$ or $\frac{3}{4}$

4. Which is greater? $\frac{3}{8}$ or $\frac{1}{3}$

Analysis of children's task

Problem	Strategies Elicited	Connections to CCSSM
1. 7/10 vs. 3/5	Common Denominators/Same-Size-Pieces 3/5 = 6/10. Because 6/10 represents fewer pieces of size 1/10 than 7/10 does, 6/10 < 7/10.	3.NF.3d 4.NF.2
2. 7/8 vs. 9/10	Comparing to a Benchmark of 1 7/8 is 1/8 less than 1; 9/10 is 1/10 less than 1. Because eighths are larger pieces than tenths, 1/8 > 1/10. So 7/8 is "missing more" than 9/10; thus 7/8 < 9/10.	3.NF.3d 4.NF.2
3. 4/3 vs. 3/4	Comparing to a Benchmark of 1 Given that 3/3 = 1, 4/3 > 1; given that 4/4 = 1, 3/4 < 1. Then 4/3 > 1 > 3/4, which means 4/3 > 3/4.	4.NF.2
4. 3/8 vs. 1/3	Common Numerators/Same-Number-of-Pieces 1/3 = 3/9. 3/8 and 3/9 each have the same number of pieces (3). Because eighths are larger pieces than ninths, 3/8 > 3/9.	3.NF.3d 4.NF.2

Modifications to Increase Cognitive Demand

Discourage familiar, algorithmic procedures

PSTs mainly rely on common denominators & converting to decimals/percents
(Livy, 2011; Olanoff, Lo, & Tobias, 2014; Yang, Reys, & Reys, 2009)

Develop multiple fraction comparison strategies based on reasoning

Create opportunities to reason about:

- fractions greater than one
- benchmark values other than one

Modifications of problems

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1) 7/10 vs. 8/9
2) 8/9 vs. 12/13
3) 24/7 vs. 34/15
4) 3/7 vs. 6/11

Modifications of problems

Distance from a Benchmark Value – BVD [1]

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Opportunity to discuss children's "gap thinking"

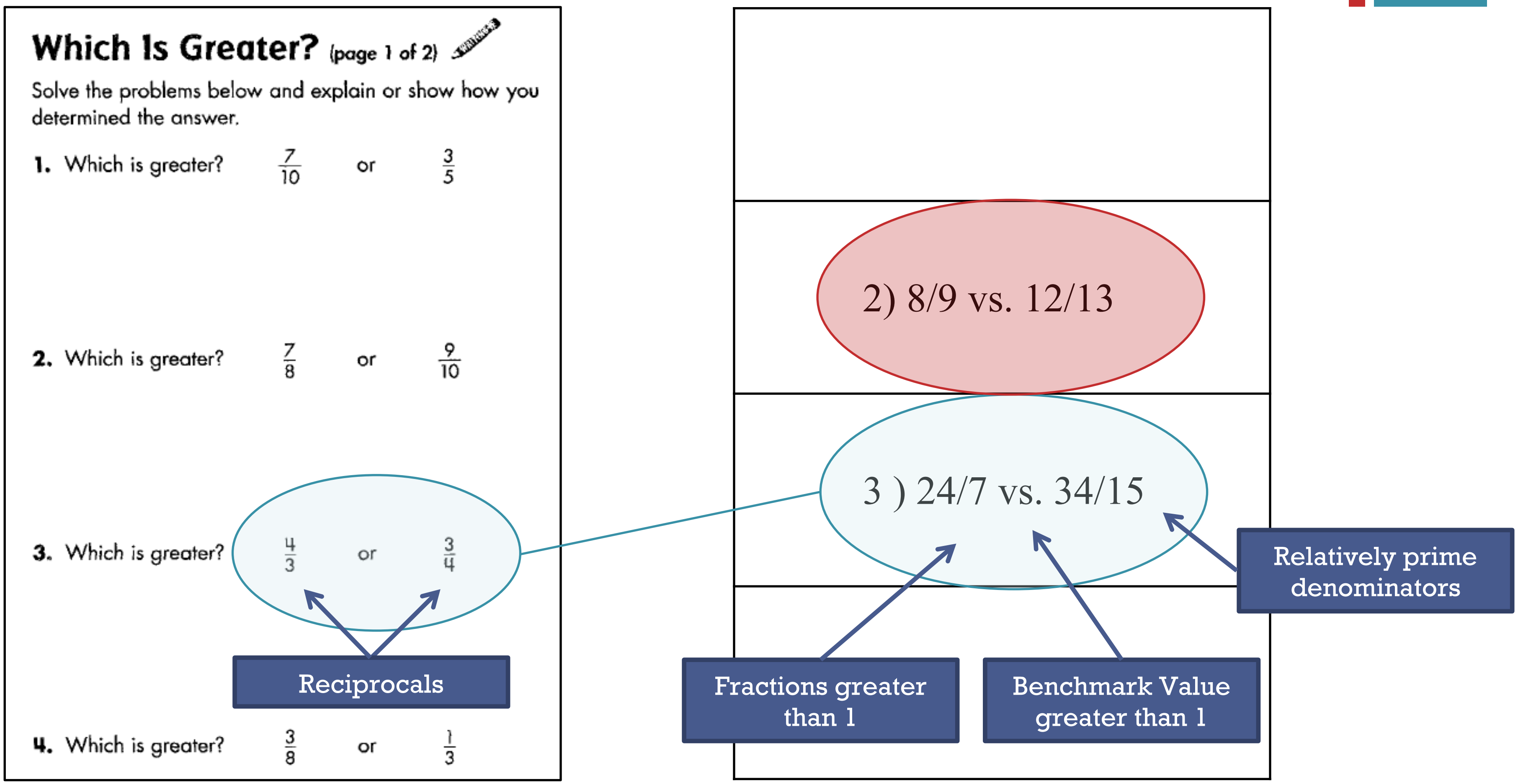
2) 8/9 vs. 12/13

Relatively prime denominators

Discourage percents, decimals, and reliance on visual models

Modifications of problems

Benchmark Value Between – BVB [3]



Modifications of problems

Same Number of Pieces - SNP

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Benchmark Value Between – BVB [1/2]

2) 8/9 vs. 12/13

3) 24/7 vs. 34/15

4) 3/7 vs. 6/11

Multiple strategies

Fractional Benchmark Value (1/2)

Modifications of problems

Greater Number of Larger Pieces - GLP

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Encourage additional strategy: GLP

1) 7/10 vs. 8/9

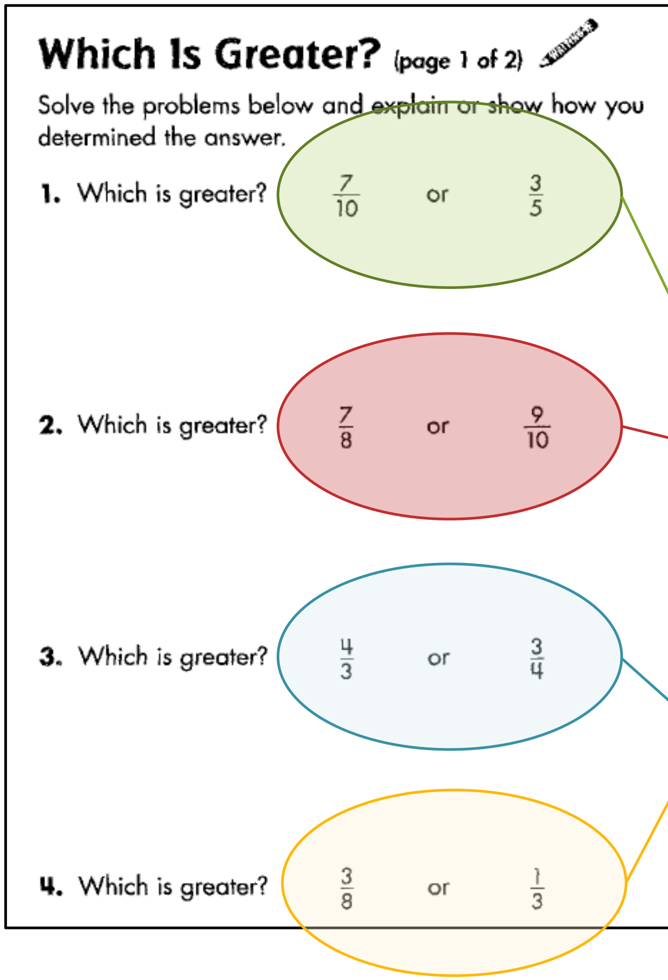
2) 8/9 vs. 12/13

3) 24/7 vs. 34/15

4) 3/7 vs. 6/11

Discourage the use of percents and decimals

Modifications of problems



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Fraction Comparison	Target strategy
1) 1/2 vs. 17/31	BV [1/2], SSP, SNP
2) 2/17 vs. 2/19	SNP
3) 4/7 vs. 9/14	SSP
4) 3/7 vs. 6/11	SNP or BV [1/2]
5) 8/9 vs. 12/13	BVD [1]
6) 13/15 vs. 17/19	BVD [1]
7) 5/6 vs. 6/5	BVB [1]
8) 7/10 vs. 8/9	GLP
9) 1/4 vs. 25/99	BV [1/4] or SNP
10) 24/7 vs. 34/15	BVB [3]

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Modifications of instructions

Which Is Greater? (page 1 of 2)

Solve the problems below and explain or show how you determined the answer.

"In this task, children compare fractions using methods of their choice, including but not limited to drawing pictures, converting to percents, and finding common denominators."

Discourage the use of familiar procedures

For each set of fractions below, circle the fraction that is greater, or if the fractions are equivalent, write "=" in between them. For each comparison give an explanation, **other than converting to common denominators**, for why the circled fraction is greater (or why the fractions are equivalent). Please make sure that you can **explain each comparison in a "sense-making" fashion.**

Calculators may not be used on this task.

Discourage converting fractions to percents and decimals

Promote "sense-making"

Data collection

Setting (n=61)

- 3 researchers as instructors
- 3 institutions
- 4 undergraduate mathematics content courses

Enactment

- Worked in groups during class time
- Collected PSTs' written work prior to class discussion

Results (n=61)

Fraction Comparison	Target strategy	# of PSTs who answered (n=61)	% of PSTs who answered correctly*	% of PSTs who used the target strategy*	Responses using common denominators (%)	Responses using conversions to decimals/percents (%)
1) 7/10 vs. 8/9	GLP	52 (85%)	98%	6%	10%	10%
2) 8/9 vs. 12/13	BVD [1]	53 (87%)	85%	68%	0%	8%
3) 24/7 vs. 34/15	BVB [3]	43 (70%)	95%	77%	2%	5%
4) 3/7 vs. 6/11	SNP; BVB [1/2]	59 (97%)	98%	8%; 58%	3%	2%

*Percentages based on students who answered the problem

Results (n=61)

At least 85% of answers were correct

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Least used strategy

Most did not use an explicit strategy:
7/10 = 0.7 & 8/9 is close to 1

Frequent use of familiar procedures

One PST used BVB [3/4]

*Percentages based on students who answered the problem

Results (n=61)

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Least successfully answered

Discouraged use of familiar procedures

*Percentages based on students who answered the problem

Results (n=61)

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Answered by fewest number of PSTs

Most successful at eliciting target strategy

Discouraged use of familiar procedures

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More use of BVB than SNP

2 PSTs used BVD [1]

Discouraged use of familiar procedures

*Percentages based on students who answered the problem

Discussion of Goals

Goal 1: Discourage familiar algorithmic procedures

1) 7/10 vs. 8/9

90 was an easy-to-find common denominator

7/10 easily converted to decimal

Problem needs adjustment

2) 8/9 vs. 12/13

3) 24/7 vs. 34/15

4) 3/7 vs. 6/11

Successfully discouraged common denominators & conversion to percents and decimals (5-8%)

Discussion of Goals

Goal 2: Develop multiple fraction comparison strategies

1) 7/10 vs. 8/9

2) 8/9 vs. 12/13

3) 24/7 vs. 34/15

4) 3/7 vs. 6/11

Greater Number of Larger Pieces did not emerge naturally (6%)

Successfully elicited multiple *Benchmark Value* strategies from the majority of PSTs

- *Benchmark Value Between*
- *Distance from a Benchmark*

Same Number of Pieces used rarely (8%)

Discussion of Goals

Goal 3: Successfully reason about fractions greater than one and with benchmark values other than one

1) 7/10 vs. 8/9
2) 8/9 vs. 12/13
3) 24/7 vs. 34/15
4) 3/7 vs. 6/11

Benchmark Value Between [3]:
95% responses correct

Benchmark Value Between [1/2]:
Used by majority

Task Masters

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email: masters@mathtaskmasters.com