Facilitating prospective teachers' fraction number sense development through problem solving and problem posing

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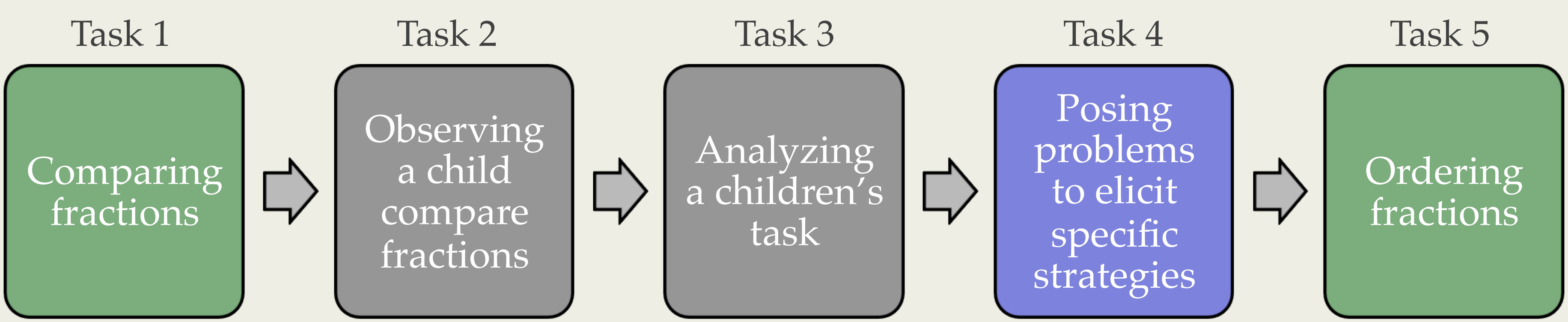
Statement of the problem

- Rational number is a major focus in the elementary grades
- **Rational number sense** is important (e.g., Lamon, 2012) yet challenging for children (e.g., Behr et al., 1984) and teachers (e.g., Yang, Reys, & Reys, 2008)
- While much research has focused on children's and teachers' *current* knowledge, little has examined how to help them *move beyond* their current knowledge (Thanheiser, et al., 2014)

Purpose of the study & context

- Examine prospective teachers' learning from their participation in a task sequence designed to help them develop rational number sense by creating and developing a deep understanding of sense-making strategies for comparing fractions identified in the literature (e.g., Behr, et al., 1984; Lamon, 2012; Van de Walle, 2004)

- Task sequence included ***problem solving tasks***, ***practice-based tasks*** (Smith, 2001), and ***problem posing tasks*** (Crespo, 2003; Silver, 1994)



see Tobias, et al., 2014 for details on design process of task sequence

Methodology

- 23 prospective elementary teachers
- Task sequence implemented approximately halfway into the semester
- Teachers' written work was collected at following points in the task sequence:

Task 1	small group quiz	Task 4 (hmwk)	homework	small group quiz	homework	exam	final exam
Day 1	*Day 3*	*due Day 4 (assigned Day 3)*	*due Day 6 (assigned Day 4)*	*Day 5*	*due Day 7 (assigned Day 6)*	*Day 8*	*one month after Day 8*

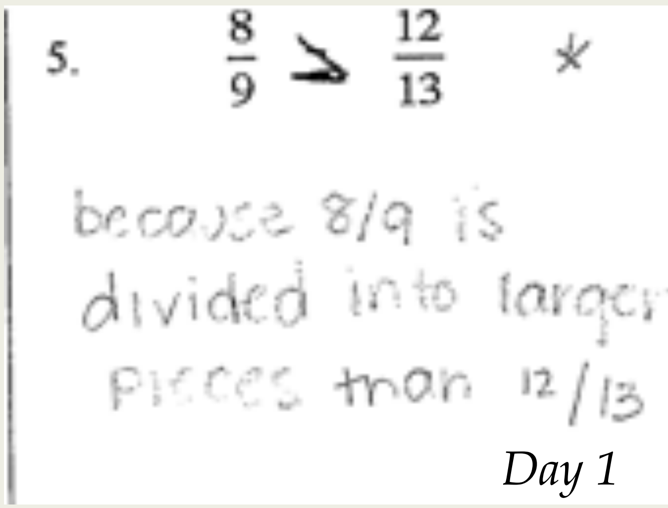
Selected results:
Focusing on the "benchmark distance" strategy

Example of a problem that elicits this strategy:
Compare 8/9 and 12/13

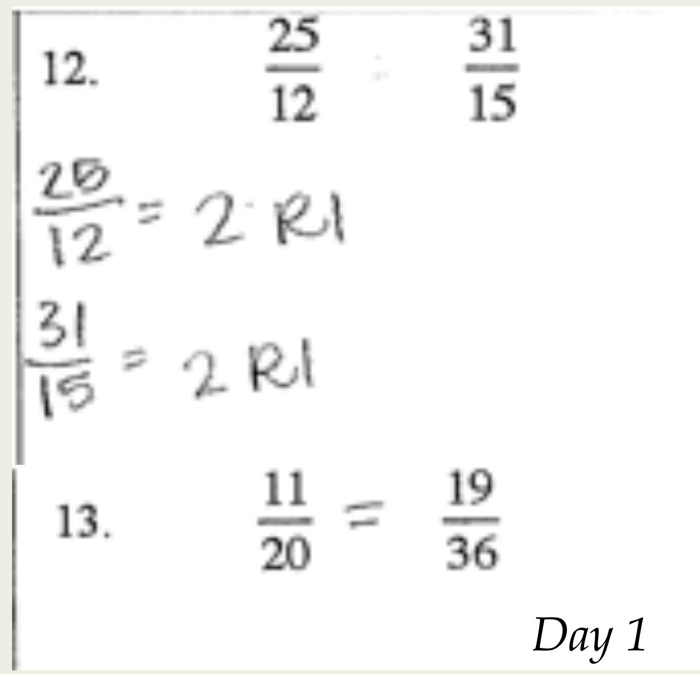
- To compare these fractions without the use of common denominators (or converting to decimals or percents) requires considering:
  - the original fractions
  - the distance each of these fractions is from the benchmark
  - how these distances compare
  - how the difference in these distances affect the size of the original fractions

Selected results:
1. Misconceptions disappeared over time

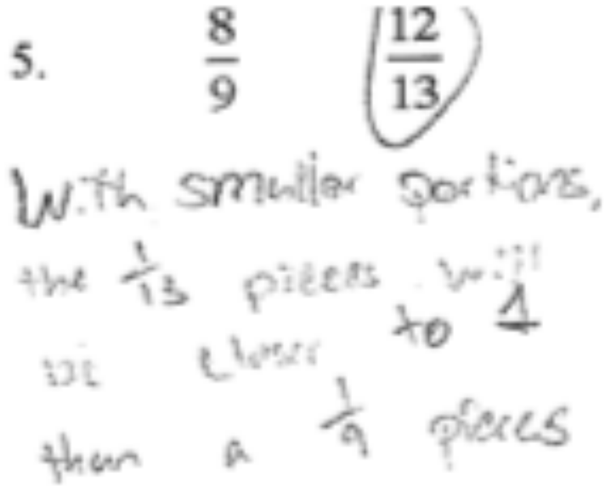
Example 1:
not attending to both the size of the pieces AND the number of pieces



Example 2:
gap thinking



Selected results:
2. Argument quality improved over time



Day 1

- Identifies greater fraction
- Appears to use benchmark distance strategy
- Argument is not clear and would not convince others – e.g., others would wonder: what do the 1/13 and 1/9 mean? Isn't 1/13 actually further away from 1 than 1/9?

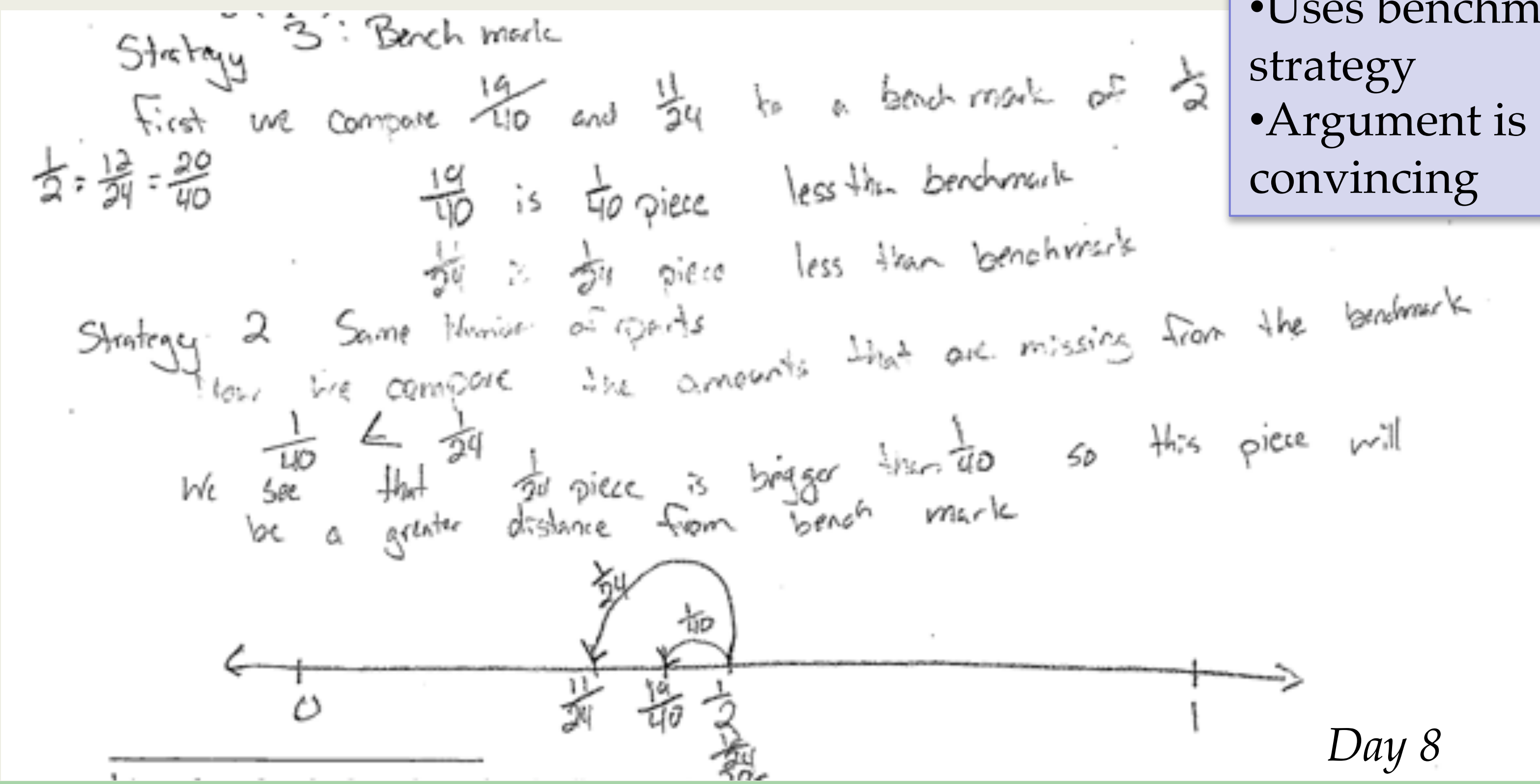
11/18 vs 13/21

"The bench mark is 2/3. 11/18 is 1/18 away from 2/3 and 13/21 is 1/21 away from 2/3/ Since a 1/21 is smaller, it is a short distance away from the benchmark. Since the bench mark is bigger than each number, this means 13 1/21 will be closer to the 2/3 benchmark and it is also the bigger number."

Day 4

- Identifies greater fraction
- Uses benchmark distance strategy
- Argument is clearer and more convincing – e.g., 1/18 and 1/21 are given meaning

Selected results:
2. Argument quality improved over time



- •Identifies greater fraction
- •Uses benchmark distance strategy
- •Argument is clear and convincing

Day 8

Selected results:
3. Teachers posed problems that elicited the strategy they intended

- Selecting/designing examples that serve particular purposes is central to the work of teaching, yet challenging (e.g., Zaslavsky, yesterday morning)
- Homework assignment due on Day 4…
  - 13 teachers (~60%) posed 14 problems in which they intended to elicit the benchmark distance strategy
    - 12 of the teachers' problems elicited this strategy (as determined by "experts" solving the problems)
- On the final exam…
  - 17 teachers posed at least one problem in which they intended to elicit the benchmark distance strategy – 6 of which did not do so on the homework
    - These problems elicited their intended strategy

Discussion

- Teachers' understanding of the benchmark distance strategy improved over time, as evidenced by their work on problem solving and problem posing tasks
- Engaging in problem solving and problem posing provided opportunities for teachers to develop different aspects of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008)
- Our future work will involve examining all of the problem solving and problem posing data collected during this implementation

Thank you for coming!

The task sequence that is the focus of this presentation, a facilitation guide, and these presentation slides are available on our website:

www.mathtaskmasters.com

You can contact us our individual email addresses or at:
masters@mathtaskmasters.com