


## Task Design in Mathematics Content Courses for Preservice Elementary Teachers: A Collaborative Approach

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### I. Original Children's Task from *Investigations in Number, Data, and Space* (Russell et al., 2008)

### Which Is Greater?

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Solve the problems below and explain or show how you determined the answer.

- Which is greater?  $\frac{7}{10}$  or  $\frac{3}{5}$
- Which is greater?  $\frac{7}{8}$  or  $\frac{9}{10}$
- Which is greater?  $\frac{4}{3}$  or  $\frac{3}{4}$
- Which is greater?  $\frac{3}{8}$  or  $\frac{1}{3}$

## II. Initial Task Design for PSTs (Version #1 used Spring 2013)

### IN-CLASS TASK

\*Calculators may not be used on this task.\* For each set of fractions below, circle the fraction that is greater, or if the fractions are equivalent, write “=” in between them. For each comparison give an explanation for why the circled fraction is greater (or why the fractions are equivalent). Please make sure that you can explain each comparison in a “sense-making” fashion.

1) $\frac{1}{2}$ $\frac{17}{31}$	2) $\frac{2}{17}$ $\frac{2}{19}$	3) $\frac{4}{7}$ $\frac{9}{14}$	4) $\frac{3}{7}$ $\frac{6}{11}$
5) $\frac{8}{9}$ $\frac{12}{13}$	6) $\frac{13}{15}$ $\frac{17}{19}$	7) $\frac{5}{6}$ $\frac{6}{5}$	8) $\frac{7}{10}$ $\frac{8}{9}$
9) $\frac{1}{4}$ $\frac{25}{99}$	10) $\frac{24}{7}$ $\frac{34}{15}$	11) $\frac{2}{7}$ $\frac{3}{8}$	

### HOMEWORK

1. As a teacher you will often need to create problems for your students with a particular goal in mind. Imagine your goal is to elicit some of the strategies for fraction comparison we learned today. To this end:

- a) Complete the following table by creating three fraction comparison problems for a classmate that would be most efficiently solved using strategies we learned in class. Each problem must target a **different** strategy. For each problem, specify the strategy that you are targeting and provide a sample solution and justification of your selected strategy and solution.

Fractions that are being compared	Strategy I am targeting	Sample solution ( <i>including an explanation/justification</i> ) using my targeted strategy
1.		
2.		
3.		

- b) What other strategies might be used to solve your three problems? Try to anticipate various alternative strategies your partner may use.

*For example, Amy created the problem, “Compare  $\frac{8}{7}$  and  $\frac{9}{6}$ ” thinking that her partner would use a benchmark strategy to compare both fractions to 1 (i.e.,  $\frac{8}{7}$  is  $\frac{1}{7}$  larger than 1 and  $\frac{9}{6}$  is  $\frac{3}{6} = \frac{1}{2}$  larger than one. Since  $\frac{1}{2}$  is larger than  $\frac{1}{7}$ , then  $\frac{9}{6}$  must be larger than  $\frac{8}{7}$ ). However, Amy’s partner stated that  $\frac{9}{6}$  has more pieces than  $\frac{8}{7}$  and the pieces are larger, thus it must be larger.*

- c) Are there ways to modify the problems you wrote to discourage your partner from using strategies besides the one you intended?

2. Consider the page from the *Investigations* curriculum for fifth grade (shown on p. 1). Which of our strategies do you think each problem is meant to elicit? Solve each problem using the strategy that you think it is meant to elicit.

### III. Second Task Design for PSTs (Version #2 used Fall 2013)

#### IN-CLASS TASK

For each set of fractions below, circle the fraction that is greater (or if the fractions are equivalent, write “=” in between them), and provide a “sense-making” explanation for how you know. You may use pictures if that is helpful to you, but your explanation cannot rely solely on a picture.

Notes: *Calculators may not be used. Feel free to work on these problems in any order that makes sense to you. If you find yourself struggling with any of the problems, skip them and revisit them later.*

1) $\frac{1}{2}$ $\frac{17}{31}$	2) $\frac{2}{17}$ $\frac{2}{19}$	3) $\frac{4}{7}$ $\frac{9}{14}$	4) $\frac{3}{7}$ $\frac{6}{11}$
5) $\frac{8}{9}$ $\frac{12}{13}$	6) $\frac{13}{15}$ $\frac{17}{19}$	7) $\frac{15}{17}$ $\frac{19}{18}$	8) $\frac{7}{10}$ $\frac{8}{9}$
9) $\frac{1}{4}$ $\frac{25}{99}$	10) $\frac{24}{7}$ $\frac{34}{15}$	11) $\frac{2}{7}$ $\frac{3}{8}$	12) $\frac{25}{12}$ $\frac{31}{15}$
13) $\frac{11}{20}$ $\frac{19}{36}$	14) $\frac{2}{9}$ $\frac{3}{8}$	15) $\frac{18}{25}$ $\frac{16}{27}$	

#### HOMEWORK

- As a teacher you will often need to create problems for your students with a particular goal in mind. Imagine that your goal is to elicit some of the strategies for fraction comparison that we have co-created over the last few classes. Specifically, create fraction comparison problems for a classmate that would have the features described in the left-most column of the table below:

Design a problem that...			Sample solution ( <i>including an explanation/justification</i> ) using my targeted strategy
...can be solved with Strategy #3 (benchmark). <i>Choose a benchmark other than <math>\frac{1}{2}</math> or 1, since we have used those as benchmarks several times already.</i>			
...can be solved with Strategy #4 (greater number of larger pieces).			
...requires using a combination of two or more of our strategies.			
As we discussed in class this morning, your given fractions should be already be in simplest form, and, you should choose your fractions so that no other strategy other than the one you're targeting would be especially easy/obvious to a classmate!			

- Consider the page from the *Investigations* curriculum for fifth grade (shown on p. 1). Which of our strategies do you think each problem is meant to elicit? Solve each problem using the strategy that you think it is meant to elicit.