Greater Number of Larger Pieces: A Strategy to Promote Prospective Teachers' Fraction Number Sense Development

TaskMasters

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Fractions as Measures

- Develop students' and teachers' familiarity with a variety of fraction interpretations, rather than focus solely on the traditionally-taught *part-whole* interpretation (Kieren, 1976; Lamon, 2012; Thompson & Saldanha, 2003).
- Both the Common Core and International Curricula support using the measure interpretation of fractions in order to overcome the limits of the traditional part-whole interpretation.

Fractions as Measures

CCSS content standard (3.NF.A.1): *Understand a fraction 1/*b *as the quantity formed by 1 part when a whole is partitioned into* b *equal parts; understand a fraction* a/b *as the quantity formed by* a *parts of size 1/*b.

- Supports children's development of fraction addition and subtraction knowledge (Son, Lo, Watanabe, 2015)
- Helps students avoid common errors (e.g., 3/7 + 2/7 = 5/14) (Mack, 1995).
- Ideas can be extended to addition and subtraction of fractions with unlike denominators (McNamara, 2015).

Designing a Task

Task Goal

Shift PTs' perspective on fractions from a *part-whole* to a *measure* interpretation, and in doing so, begin to see fractions as quantities.

Task Enactment

Provide learners with repeated opportunities to grapple with problems and generate their own solution strategies instead of apply a strategy made explicit by an instructor.

Designing a Task

- Ten fraction comparison problems
- Attend to several fraction comparison strategies:
  - Common denominators (i.e., "same size pieces")
  - Common numerators (i.e., "*same number pieces*")
  - Comparing to a benchmark value
  - Greater number of larger pieces (GLP)
- PTs work in groups to solve each problem, and then share out strategies as a whole class

Greater Number of Larger Pieces (GLP)

- Use the measure interpretation of fractions to consider each fraction as a certain number of equal-sized pieces.

- Requires the simultaneous coordination of two quantities - one referring to the *number* of fractional pieces and one referring to the *size* of those pieces.

  Example: 18/25 to 16/27
  18/25 → 18 fractional pieces each of size 1/25
  16/27 → 16 fractional pieces each of size 1/27

  Pieces of size 1/25 are larger than pieces of size 1/27 *and* there is a greater number of pieces of size 1/25 (18 > 16). Therefore, 18/25 > 16/27.

Task Directions

- **Task Version #1:** For each set of fractions below, circle the fraction that is greater, or if the fractions are equivalent, write "=" in between them. For each comparison, give an explanation, other than converting to common denominators, for why the circled fraction is greater or why the fractions are equivalent. Calculators may not be used on this task.

Task Version 1

Problem	Fractions to Compare		Problem	Fractions to Compare
#1	1/2 vs. 17/31		#6	13/15 vs. 17/19
#2	2/17 vs. 2/19		#7	5/6 vs. 6/5
#3	4/7 vs. 9/14		#8	7/10 vs. 8/9
#4	3/7 vs. 6/11		#9	1/4 vs. 25/99
#5	8/9 vs. 12/13		#10	24/7 vs. 34/15

Data collection

Setting (n=61)

- 3 researchers as instructors
- 3 institutions
- 4 undergraduate mathematics content courses

Enactment

- PTs instructed not to use common denominators or calculators
- Worked in groups during class time
- Collected PTs' written work prior to class discussion

Launching the Task

- Prompts for small group work, followed by whole class discussion:
  - List everything you know about 7/8.
  - Keeping the *denominator* the same, find 3 fractions that are greater than 7/8, and 3 fractions that are less than 7/8.
  - Keeping the *numerator* the same, find 3 fractions that are greater than 7/8, and 3 fractions that are less than 7/8.
- Goal is to help PTs begin the transition towards interpreting fractions as measures.
  - E.g., 7/8 > 5/8 because 7/8 has more pieces of 1/8.
  - E.g., 7/8 > 7/9 because 7/8 is eighths are larger pieces than ninths, and 7 bigger pieces is greater than 7 smaller pieces.

What happened?

- What do you think happened?
- Where are opportunities to develop GLP?

Results Task Version 1 (n=61)

Fraction Comparison	Target strategy	# of PTs who answered (n=61)	% of PTs who answered correctly*	% of PTs who used the target strategy*	Responses using common denominators (%)	Responses using conversions to decimals/percents (%)
1) 7/10 vs. 8/9	GLP	52 (85%)	98%	6%	10%	10%

- Only 26% of people used “valid” strategies.
- The remaining 74% of responses offered incorrect or incomplete reasoning
- Additionally, the three PTs who developed GLP were in only two of the four classes; thus the eventual presentation of the strategy in the other two classes had to come from the instructors, as opposed to the knowledge being constructed and shared by the learners.

Task modifications

- We made a number of modifications to the task, in part to help elicit the GLP strategy.
- First, we added 2/9 vs 3/8 (problem #14), which can be solved using a variety of strategies including the GLP strategy.
- Second, we added 2/7 vs 3/8 (problem #11), which is purposefully similar to 2/9 vs 3/8, but cannot be solved using GLP.
- Third, we added 18/25 vs 16/27 (problem #15).
- A total of five new items were added to the task, resulting in a second iteration containing 15 problems. Problem #8, the original GLP problem (7/10 vs. 8/9), was left unmodified.
- Additionally, we put further emphasis on probing students' explanations during the launch, no longer accepting things like $7/9 < 7/8$ because it is further from 1.

What happened?

- What affordances do you think the modifications provided in relation to GLP?

Problem	Fractions to Compare
8	7/10 vs. 8/9
14	2/9 vs 3/8
15	18/25 vs 16/27

- What do you think happened this time?

Task Version #1 and #2 Results (Task 1, n=61; Task 2, n=63)

Iteration	Problem	GLP Problem	# of PTs who answered the question	% of responses received with correct answers	% of responses received using GLP
1	#8	7/10 vs 8/9	52	98.0%	6.0%
2	#8	7/10 vs 8/9	61	96.7%	14.8%
2	#14	2/9 vs 3/8	52	96.2%	21.2%
2	#15	18/25 vs 16/27	46	95.7%	41.3%

Results/Discussion

- The data provide evidence that the second iteration of the task was more successful in eliciting the GLP strategy. Out of the 63 PTs who worked on the second iteration of our task, 21 of them (33%) used GLP on at least one problem, 19% used it on two, and 8% used it on all three of the applicable problems.

- One task modification strategy that has potential for supporting PT learning is creating problems like Problem #15 that lend themselves to particular solution strategies while discouraging the use of alternate strategies. In attempting to solve this problem, PTs were forced to abandon certain strategies that may not support reasoning and sense making.

Conclusions

- Our work supports a **nuanced approach to task design** that makes use of both problems that elicit multiple strategies and those that narrow the field of possible solution strategies to those under investigation.

- GLP is challenging because it requires **coordinating two quantities**: number of pieces *and* size of pieces
  - 15% of PTs exhibited partial evidence of GLP reasoning on problem #15 (e.g., attending to size of pieces but not to number of pieces, or vice versa)

- Additional data on final exams show evidence of learning
  - 78.6% (44 out of 56) of PTs were able to correctly justify why the GLP strategy cannot be used to compare 27/29 vs. 31/33
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