Moving Beyond Common Denominators: Comparing Fractions Using Reasoning and Sense-Making

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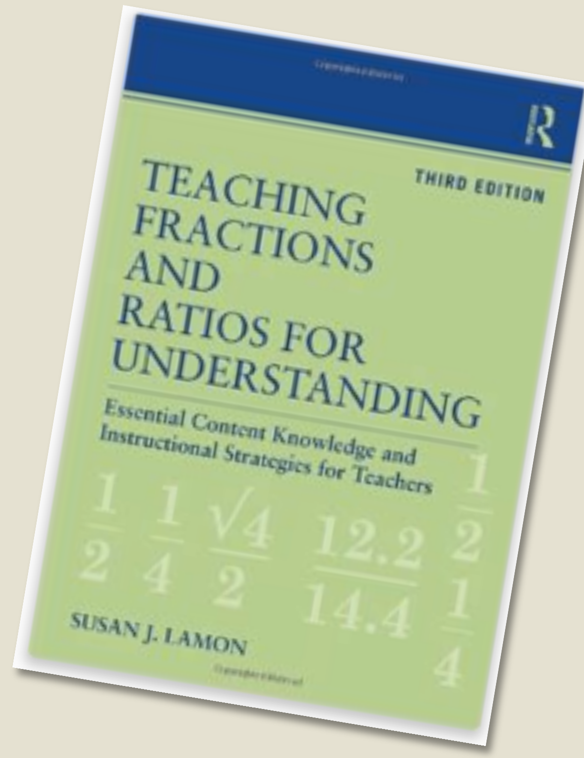
Session overview

- Brief background
- Journey through 3 tasks
- Look across tasks and consider potential for developing number sense and reasoning

Number sense and reasoning

"…students should develop an intuition that helps them make appropriate connections, determine size, order, and equivalence, and judge whether answers are or are not reasonable."

(Lamon, 2012, p. 136)



Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

NCTM, 2014, p. 10

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Implement Tasks that Promote Reasoning and Problem Solving

Effective teaching of mathematics:

- provides opportunities for students to engage in solving and discussing tasks;
- uses tasks that promote inquiry and exploration and are meaningfully connected to concepts;
- uses tasks that allow for multiple entry points; and
- encourages use of varied solution strategies.

NCTM, 2014, p. 17

Build Procedural Fluency from Conceptual Understanding

Effective teaching of mathematics:

- builds on a foundation of conceptual understanding;
- results in generalized methods for solving problems; and
- enables students to flexibly choose among methods to solve contextual and mathematical problems.

NCTM, 2014, p. 42

Build Procedural Fluency from Conceptual Understanding

"To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. ***Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.***"

Martin (2009, p. 165)

Sorting fractions: Task #1

Identify some fractions that would fit in the following buckets:

close to 0 | close to ½ | close to 1

Consider:

- In what ways could your fractions be further sorted?
- What other fractions and/or buckets could be included in order to…
  - uncover various ways of thinking (both correct and incorrect)?
  - invite creative thinking?
  - be especially challenging?

Adapted from Sowder, Sowder, & Nickerson, 2010

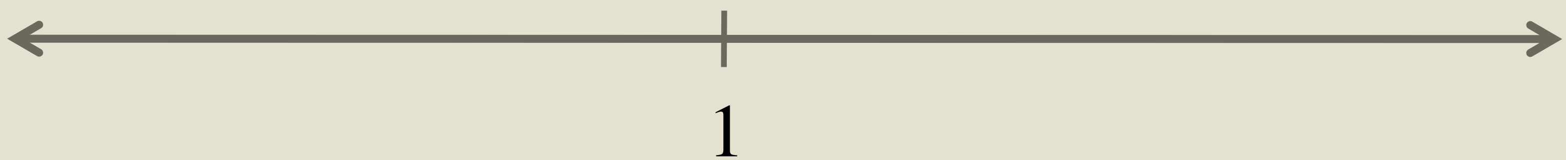
Comparing fractions: Task #2

- Try to compare the fractions using reasoning
- Consider multiple reasoning strategies if you have time
- In what ways would the learners with whom you work approach each problem?

Reasoning strategies

Comparing to a benchmark (e.g., problem g)

If one fraction is larger than our benchmark and the other fraction is smaller than our benchmark, the fraction that is larger than our benchmark is greater than the fraction that is smaller than our benchmark.



$\frac{17}{17}$

$\frac{18}{18}$

here the benchmark is **<u>between</u>** the fractions being compared

Reasoning strategies

Comparing to a benchmark (e.g., problem e)

If both fractions are larger (or smaller) than our benchmark, then we need to consider how much more (or less) each fraction is than our benchmark in order to determine the greater fraction.

here the benchmark is larger/smaller than both of the fractions being compared, so **their distances from the benchmark** must be considered

Lamon, 2012

Reasoning strategies

$$\frac{2}{17} \qquad \frac{2}{19}$$

Same number of parts (e.g., problem b)

When comparing fractions in which the same number of parts are being considered and the parts are different sizes, the fraction with the larger-size parts has the greater value

Lamon, 2012

Essential understanding

3.NF.A.1

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Why does the common denominator strategy make sense?

$$\frac{8}{14} \qquad \frac{9}{14}$$

Same size parts

When comparing fractions with the same-size parts, the fraction with the greater number of parts being considered has the greater value

Another reasoning strategy

$$\frac{18}{25} \qquad \frac{16}{27}$$

Greater number of larger parts (e.g., problem o)

If you have *more* parts that are *larger-sized*, and you're comparing that to having *fewer* parts that are *smaller-sized*, the fraction that has more parts that are larger-sized has the greater value.

(But if you have *fewer* parts that are *larger-sized* and you're comparing that to having *more* parts that are *smaller-sized*, then you cannot determine which fraction has the greatest value, without using some other strategy).

Connecting to CCSS

3NF.A.3.D

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.A.2

Compare two fractions with different numerators and different denominators, **e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.** Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Connecting to CCSS

SMP 3:

Construct viable arguments and critique the reasoning of others

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions…Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Connecting to curricula

Which is greater? $\frac{7}{10}$ or $\frac{3}{5}$

Which is greater? $\frac{7}{8}$ or $\frac{9}{10}$

Which is greater? $\frac{4}{3}$ or $\frac{3}{4}$

Which is greater? $\frac{3}{8}$ or $\frac{1}{3}$

Choose two pairs of fractions from the following list. Use pictures, numbers, and/or words to find two ways to show which fraction is greater and to explain how you know.

$\frac{1}{3}$ and $\frac{1}{4}$	$\frac{1}{2}$ and $\frac{3}{5}$	$\frac{5}{8}$ and $\frac{7}{10}$	$\frac{3}{2}$ and $\frac{4}{3}$
$\frac{9}{5}$ and $\frac{7}{4}$	$\frac{2}{3}$ and $\frac{5}{6}$	$\frac{1}{8}$ and $\frac{2}{10}$	$\frac{3}{4}$ and $\frac{4}{5}$

Find three ways to show that $\frac{7}{8}$ is greater than $\frac{5}{6}$. Use pictures, numbers, and/or words.

Russell, et al., 2008

Ordering fractions: Task #3

- Which reasoning strategies might emerge when students order these fractions?
- What other fractions would you want to include in this set, in order to push students' thinking?
- What extension questions would you want to pose? Why?

$\frac{1}{200}$ $\frac{1}{99}$ $\frac{12}{11}$ $\frac{1}{4}$ $\frac{4}{3}$ $\frac{10}{5}$

$\frac{5}{2}$ $\frac{24}{24}$ $\frac{5}{4}$ $\frac{3}{4}$ 1 0

Take a few minutes to consider…

- In what situations is the common denominator strategy *especially* useful? In what situations might an alternate strategy be *more* useful?
- How might you adapt this sequence of tasks to meet the needs of the learners with whom you work?
  - What challenges might you face in implementing this sequence of tasks?
  - What struggles might learners encounter? How might you support them through these struggles?
  - What task(s) might you use to follow this sequence of tasks?

Thank you for coming!

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The handout used in this session will be uploaded to the Speaker's Corner. The slides and other materials from this session will also be on our website: www.mathtaskmasters.com.

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