

Fraction Learning Trajectories in Content Courses for Prospective K-8 Teachers

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Session Overview

- Provide a description of learning trajectories in mathematics education research and the CCSS-M
- Discuss our fraction comparison task and results from a study focusing on fraction comparison
- Examine PSTs' work on fraction comparison to propose a learning trajectory
- Provide implications for content courses and research in teacher education

Learning Trajectories in Education

- Hypothetical Learning Trajectory (Simon, 1995).
- Common Core State Standards (CCSSO 2010)
- Fraction Learning Trajectories for PSTs
 - Tobias (2009, 2013)
 - Tobias (2013) found that PSTs' fraction understanding may not develop as linearly as one might expect.
 - Wilson, Mojica, & Confrey (2013)

Fraction Learning Trajectory for PSTs (Tobias, 2009)

Phase	Topic
One	Partitioning
	Unitizing
	Equivalence
Two	Comparing
	Ordering
Three	Addition
	Subtraction
Four	Multiplication
	Division

CCSS-M: Fraction Comparison

Third Grade

- Compare two fractions with the same numerator or same denominator by reasoning about their size

Fourth Grade

- Compare two fractions with different numerators and different denominators, e.g. by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

Fraction Comparison Task

1. **Comparing Fractions**
2. Observe a Child Compare Fractions
3. Analyze a Children's Task
4. Posing Problems to Elicit Specific Strategies
5. Ordering Fractions

Learning Goals

1. PSTs will develop the following reasoning strategies to compare and order:

- Same Size Pieces
- Same Number of Pieces
- Benchmark Comparison
- Greater Number of Larger Pieces

2. PSTs will:

- Have a general sense of the size of a fraction
- Identify which comparison strategies would be appropriate for a given situation

Launch

- Make a list of everything you know about the number $\frac{7}{8}$.
- Keeping the denominator the same, find 3 fractions that are greater than $\frac{7}{8}$, and find 3 fractions that are less than $\frac{7}{8}$.
- Keeping the numerator the same, find 3 fractions that are greater than $\frac{7}{8}$, and find 3 fractions that are less than $\frac{7}{8}$.

Launch Discussions

- What the denominator represents (size of each piece)
- What the numerator represents (number of pieces you have)
- How much larger/smaller $\frac{7}{8}$ is from $\frac{1}{2}$ and 1
- Unitizing ($\frac{7}{8} =$ seven $\frac{1}{8}$ pieces)

Fraction Comparison Task

Problem	Fractions to Compare	Intended Strategy
1	$\frac{1}{2}$ vs. $\frac{17}{31}$	Benchmark Value Equivalent to $\frac{1}{2}$ Equivalent Fractions – Same Sized Pieces Equivalent Fractions – Same Number of Pieces
2	$\frac{2}{17}$ vs. $\frac{2}{19}$	Same Number of Pieces
3	$\frac{4}{7}$ vs. $\frac{9}{14}$	Equivalent Fractions – Same Sized Pieces
4	$\frac{3}{7}$ vs. $\frac{6}{11}$	Benchmark Value Between $[\frac{1}{2}]$ Equivalent Fractions – Same Number of Pieces
5	$\frac{8}{9}$ vs. $\frac{12}{13}$	Benchmark Value Distance [1]
6	$\frac{13}{15}$ vs. $\frac{17}{19}$	Benchmark Value Distance [1]
7	$\frac{15}{17}$ vs. $\frac{19}{18}$	Benchmark Value Between [1]

Problem	Fractions to Compare	Intended Strategy
8	7/10 vs. <u>8/9</u>	Greater Number of Larger Pieces
9	1/4 vs. <u>25/99</u>	Benchmark Equivalent to 1/4 Equivalent Fractions – Same Number of Pieces
10	<u>24/7</u> vs. 34/15	Benchmark Value Between [3]
11	2/7 vs. <u>3/8</u>	Benchmark Value Between [1/3] Benchmark Value Distance [1]
12	<u>25/12</u> vs. 31/15	Benchmark Value Distance [2]
13	<u>11/20</u> vs. 19/36	Benchmark Value Distance [1/2]
14	2/9 vs. <u>3/8</u>	Benchmark Value Distance [1/3] Greater Number of Larger Pieces
15	<u>18/25</u> vs. 16/27	Greater Number of Larger Pieces

Discussion

1. What do you notice about PSTs' thinking?
 - What do they appear to understand/not understand?
2. How successful were the PSTs with using an intended strategy to solve the problem?
3. Based on the results, develop a learning trajectory for fraction comparison.

Results

Problem	# of PSTs who answered the question (n=63)	% of responses received with correct answers	% of responses received using the intended strategy
1/2 vs. 17/31	61	95.2%	73%
2/17 vs. 2/19	62	98.4%	92.1%
4/7 vs. 9/14	61	96.8%	82.5%
3/7 vs. 6/11	63	95.2%	68.3%
8/9 vs. 12/13	58	85.7%	44.4%
13/15 vs. 17/19	55	76.2%	55.5%
15/17 vs. 19/18	59	93.7%	79.4%

Problem	# of PSTs who answered the question	% of responses received with correct answers	% of responses received using the intended strategy
7/10 vs. 8/9	61	96.7%	14.8%
1/4 vs. 25/99	60	84.1%	76.2%
24/7 vs. 34/15	60	95.2%	82.5%
2/7 vs. 3/8	57	81%	6.3%
25/12 vs. 31/15	55	84.1%	66.7%
11/20 vs. 19/36	52	81%	39.7%
2/9 vs. 3/8	52	96.2%	21.2%
18/25 vs. 16/27	46	95.7%	41.3%

Trajectory for Fraction Comparison

Level	Topic	Description	Average %
1	Common Numerator Common Denominator	Same Number of Pieces Same Size Pieces	92.1%
2	Equivalent Fractions <ul style="list-style-type: none">• Same Size Pieces• Same Number of Pieces Benchmark Value Equivalent	Use equivalent fractions to find fractions with the same size/number of pieces One of the given fractions is equivalent to the benchmark fraction	74.8%
3	Benchmark Value Between	Benchmark is between the two given fractions	59.1%
4	Benchmark Value Distance	Benchmark is above or below both fractions	39%
5	Greater Number of Larger Pieces	The fraction with more larger pieces is greater	25.8%

Implications

Research

- More research is needed focusing on PSTs' fraction comparison strategies over time.

Teacher Education

- Putting comparison problems in order (according to a learning trajectory) may aid in their understanding of the topic more (i.e. have PSTs solve comparison problems with equivalent fractions before greater number of larger pieces).

Thank You For Coming



For the comparing fractions task and facilitation notes for this task,
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