Greater Number of Larger Pieces (7/10 vs. 8/9, 2/9 vs. 3/8, 18/25 vs. 16/27)

$$
\left(\frac{18}{25}\right)>\frac{16}{27} \frac{16}{25}
$$


laths are smaller thaths
and wheres less $10^{\text {th }}$
anyway so its got
to besmaller

make the parts equal is see that 16 has less parts, then when you see that the parts are smaller, you can see that $16 / 27$ is smaller.

Benchmark Value Equivalent (1/2 vs. 17/31, $1 / 4$ vs. 25/99)
$25 / 99$ is jeatur than $1 / 4$. If you multiply $1 / 4 \mathrm{ly} 3 / 25$ you get $25 / 100$. Now both fractions hove 25 pieces but 25 pieces from 99 is greater than $25 / 100$ because the 99 pieces arelayen.

$$
\begin{gathered}
\frac{1}{2}<\frac{17}{31} \\
\frac{31}{62}<\frac{34}{62}
\end{gathered}
$$ when equal denomiztor me fond, the mentor on the right is larges.

$$
\left(\frac{1}{4}\right) \quad \frac{25}{99}
$$

$$
b / c 25 \text { is } 1 / 4 \text { of }
$$ a hundred and it is less than $1 / 4$ if it is out of 99 . Therefore $1 / 4$ is bigger

Same Size Pieces/Same Number of Pieces (2/17 vs. 2/19)

$$
1 / 17>1 / 194
$$

A whole divided in to

$$
17 \text { pieces, }+2^{*} \text { Whole }
$$ divided into 19 pieces would have diff sizedrieces. $21 / 17$ pieces would be larger than 2 $1 / 19$ pieces


the pieces are bigoyer than $19^{\text {th }}$ pieces. Both have only 2 pieces, then the ore with larger pieces would be bigger.

Learning Trajectories for PSTs
Tobias, Feldman, Welder, and Olanoff
AMIE 2017
Equivalent Fractions - Same Size Pieces/Same Number of Pieces (4/7 vs. 9/14, 3/7 vs. 6/11)
$\frac{\overline{8}}{14}=\frac{4}{7}$


Benchmark Value Between ( $24 / 7$ vs. $34 / 15,15 / 17$ vs. $19 / 18,3 / 7$ vs. $6 / 11,2 / 7$ vs. 3/8)

$$
\left(\frac{24}{7}\right)=3 \frac{3}{7} \quad \frac{34}{15}=2 \frac{4}{15}
$$

When these fractions are changed to proper
fractions, it is easy to
see that $\frac{24}{7}>\frac{34}{15}$.
$\frac{24}{7}=$ over 3 while
$\frac{34}{15}=$ less than 3 .

$$
\begin{aligned}
& \quad\left(\frac{3}{7}\right) \frac{6}{11} \\
& 3 \text { can go into } \\
& 7 \text { twice, and } 6 \\
& \text { can go into } 11 \\
& \text { only once, so } \\
& 3 \text { is a } 6 \text { geiger } \% \text { age } \\
& \text { of } 7 \text { than } 6 \text { is } \\
& \text { of } 11 .
\end{aligned}
$$

Benchmark Value Distance (8/9 vs. 12/13, 13/15 vs. 17/19, 25/12 vs. 31/15, 11/20 vs. 19/36)


